Supply Chain Scheduling in a Make-to-Order Environment

Lei Wang*, Guoqing Wang
Department of Business Administration
Jinan University, Guangzhou, P.R.China
***EMAIL: wanglei5804@163.com**

Abstract: We consider a make-to-order production-distribution system with one supplier and multiple customers. A set of jobs with due dates need to be processed by the supplier and delivered to the customers upon completion. The supplier can process one job at a time without preemption. Each customer is at a distinct location and only jobs from the same customer can be batched together for delivery. We consider the single machine scheduling problems in which the jobs belong to different families. A setup time is incurred for a job if it is the first job to be processed on a machine or its processing on a machine follows a job that belongs to another family. We consider two due date related objectives: the first is to minimize the weighted sum of the maximum lateness of jobs to customers and the delivery cost. The second objective is to minimize the weighted sum of the number of late jobs and the delivery cost. We give two optimal algorithms to solve the problems.

Keywords: supply chain; scheduling ; batch delivery ; make-to-order

I. Introduction

The Production and distribution operations are two key operational functions in a supply chain. To achieve optimal operational performance in a supply chain, it is critical to integrate these two functions and plan and schedule them jointly in a coordinated manner. There are various integrated models of production scheduling and product distribution in the literature. The objective of such models is typically to optimize both customer service level and distribution cost. To learn more about research results on this aspect, the reader is referred to see Hall and Potts [1], Chen and Variraktarakis [2],Chen and Pundoor [3],Chen and Lee [4], among others.
Another line of research related to the problem under study focuses on scheduling problem with job family setup. There are extensive research results on this problem in the literature. For example, Monma and Potts [5] examined the computational complexity of some basic models for the single- and parallel-machine cases with some common scheduling objectives. Liaee and Emmons [6] reviewed scheduling theory concerning the processing of several families of jobs on single or parallel facilities. For various performance measures, they classified the different problems as NP-hard, efficiently solvable or open. For more details on this line of research, the reader is referred to the reviews by Potts and Kovalyov [7], and Allahverdi et al. [8]. However, these studies have not considered the product transportation issue.
There are only limited results on scheduling with job families and delivery coordination. Dawande et al.[9] studied conflict and cooperation issues between a manufacturer and a distributor with a focus on setup cost in production and inventory-holding cost in distribution. Cheng and Wang [10] study machine scheduling problems in which the jobs belong to different job families and they need to be delivered to customers after processing. The objective is to minimize the weighted sum of the last arrival time of jobs to customers and the delivery cost. For the problem of processing jobs on a single machine and delivering them to multiple customers, they develop a dynamic programming algorithm to solve the problem optimally. For the problem of processing jobs on parallel machines and delivering them to a single customer, they propose a heuristic and analyze its performance bound. For the same objective, Wang and Cheng [11] study the identical parallel machine scheduling with job family setup and delivery to multiple customers. They develop heuristics for the problem and analyze their performance bounds.
In this paper we consider the single machine scheduling problems in which the jobs belong to different families and they need to be delivered to multiple customers after processing. A setup time is required for a job if it is the first job to be processed on a machine or its processing on a machine follows a job that belongs to another family. The difference between models of Cheng [10] and us is that we assume that jobs of the same family belongs to a specified customers. This means that each customer orders products from the same family and different customers order different families of jobs. Processed jobs are delivered in batches to their respective customers. Each shipment incurs a delivery cost and takes a fixed amount of time. We consider two due date related objectives: the first is to minimize the weighted sum of the maximum lateness of jobs to customers and the delivery cost. The second objective is to minimize the weighted sum of the number of late jobs and the delivery cost. The remainder of this paper is organized as follows. In Section 2 we formulate the integrated scheduling model and present some optimal properties. In Section 3 we develop optimal algorithms for two problems. In the last section we give some concluding remarks.

II. Problems and Basic Properties

In this section, we define our problems mathematically and introduce some optimality properties satisfied by all the problems that we will use in later sections.

A supplier obtains f orders from f customers that are located at different sites at time 0. According to their different processing characteristics, each order i can be seen as a job family, so there are f job families at all. Let $N_i = \{J_{i1}, J_{i2}, \cdots, J_{in_i}\}$ be the job set of customer i, for $i = 1, 2, \cdots, f$, where $n_1 + n_2 + \cdots + n_f = n$. There is a single machine available for processing all the jobs. The processing time of job J_{ij} is p_{ij} ,due date is d_{ij} and preemption is not permitted. In production, if job J_{ij} is the first job to be processed on a machine or its processing on a machine follows a job from another family, then a setup time s_i is required before J_{ij} can be processed.

Once completed on the machine , each job needs to be delivered to its customer. We assume that there are an unlimited number of transport vehicles available (e.g., they are provided by a third-party logistics service firm) to deliver the processed jobs from the factory to customers at any time. Each vehicle can load at most $B \geq n$ jobs on each trip from the factory to a customer. Let t_i and c_i denote the time and cost that a vehicle travels from the factory to customer i, for $i = 1, 2, \cdots, f$ .Note that the batch transportation time and cost are independent of the batch size. For a given schedule of the problem, we define:

C_{ij} = the completion time of job $J_{ij} \in N_i$, $i = 1, 2, \cdots, f$; $j = 1, 2, \cdots, n_i$.

D_{ij} = the delivery time of job $J_{ij} \in N_i$, which is the time when job J_{ij} is delivered to the customer i.

$L_{ij} = D_{ij} - d_{ij}$. The lateness of job $J_{ij} \in N_i$.

$U_{ij} = 1$ if $L_{ij} > 0$; else $U_{ij} = 0$.

TC = the total distribution cost.

We consider the following two due date related objectives for measuring the delivery lead-time performance of the supply chain:

(i) maximum lateness of the jobs:

$$L_{\max} = \max\{L_{ij} \mid i = 1, 2, \cdots, f; j = 1, 2, \cdots, n_i\} \qquad (1)$$

(ii) total number of late jobs: $\sum_{i=1}^{f} \sum_{j=1}^{n_i} U_{ij}$

We adopt the three-field notation $\alpha|\beta|\gamma$ widely used in scheduling research (see, e.g., [12]) to denote the problems under study. We study the following two problems:

P1: $1|s_i, f|\omega L_{\max} + (1-\omega)TC$

P2: $1|s_i, f|\omega \sum_{i=1}^{f} \sum_{j=1}^{n_i} U_{ij} + (1-\omega)TC$

In the α field, 1 means that the jobs are processed on a single machine .In the β field, s_i means that the jobs may need a setup before processing since they belong to different job families , f means that processed jobs need to be delivered to one of the f customers.. The γ field is the objective function to be minimized. $\omega \in [0,1]$ is a given constant, representing the weighting factor for the customer service level, $(1-\omega)$ is the weighting factor for the delivery cost.

Hall and Potts [1] study one supplier produce and delivery products for multiple manufacturers, this is similar with our problem. But they didn't consider job families and transportation time. For maximum lateness problem we assume that late jobs are produced and delivered. The objective function is used to ensure that the due dates are achieved as closely as possible. Second, in models which minimize the number of late jobs, we assume that a job which would be late is neither produced nor delivered. This assumption is relevant where late deliveries are not accepted.

We state the following observations for problems P1 and P2.The proofs are trivial and omitted.

Observation1. For the two problems, there exists no idle time at the beginning or between any two adjacent processed jobs on each machine in an optimal solution.

Observation2. For the two problems, there is an optimal solution in which the departure time of each shipment is equal to the completion time of the last processed job in the shipment.

Observation3. For the two problems, the job finished processing earlier should be delivered no later than the subsequent processed jobs for the same customer.

In the following sections, in order to develop efficient algorithms for the problems, we only consider solutions for problems that follow Observations 1–3.

Ⅲ. Optimal Algorithms

Optimal algorithm for P1

We have the following optimal property for problem P1.

Lemma 1. For problem P1, there exists an optimal schedule such that the jobs within each family are processed and delivered in nondecreasing order of their due dates (EDD), and jobs assigned to one delivery batch are processed consecutively on the machine.

Denote the due date time sequence for family i by $d_{i1} \le d_{i2} \le \cdots \le d_{in_i}$, $i = 1, 2, \cdots, f$. Based on the optimal properties stated in lemma 1 ,we develop the following dynamic programming algorithm for P1.

Algorithm 1:

Define the function $V(l_1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; b, j)$ as the minimum value of the maximum lateness for scheduling that satisfy the following conditions:

(1) the total number of scheduled jobs is $l_1 + \cdots + l_f$,of which l_u jobs are from the top of the EDD sequence for customer u , $u = 1, 2, \cdots, f$;

(2) the number of setups of s_i for job family i is $x_i (x_i \le l_i)$, $i = 1, 2, \cdots, f$;

(3) the number of deliveries of family i is $y_i (y_i \le l_i)$, $i = 1, 2, \cdots, f$;

(4) the number of jobs in the last delivery batch is b ,and the jobs belong to family $j (1 \le j \le f)$.

Boundary condition:

$$V(0, \cdots, 0; 0, \cdots, 0; 0, \cdots, 0; 0, 0) = -\infty \qquad (2)$$

Recurrence relation:

$$V(l_1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; b, j) =$$

$$\min \begin{cases} \max\{V(l_1, \cdots, l_j - 1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; b-1, j); \\ \quad [\sum_{u=1}^{f} (x_u s_u + \sum_{k=1}^{l_u} p_{uk}) + t_j - d_{j(l_j - b+1)}]\} \qquad \text{if } b > 1 \\ \min_{1 \le i \le f} \{\max\{V(l_1, \cdots, l_j - 1, \cdots, l_f; x_1, \cdots, x_j', \cdots, x_f; \\ \quad y_1, \cdots, y_j - 1, \cdots, y_f; b', i); \\ \quad [\sum_{u=1}^{f} (x_u s_u + \sum_{k=1}^{l_u} p_{uk}) + t_j - d_{jl_j}]\}\} \qquad \text{if } b = 1 \end{cases} \qquad (3)$$

$x_j' = x_j - 1$ if $i \ne j$; $x_j' = x_j$ if $i = j$.

Optimal solution value:

$$\min_{y_1, \cdots, y_f} \{\omega V(n_1, \cdots, n_f; x_1, \cdots, x_f; y_1, \cdots, y_f; b, j) + (1-\omega) \sum_{i=1}^{f} y_i c_i\}$$

For all $1 \le y_i \le n_i$ ($i = 1, 2, \cdots, f$).

Theorem1. Algorithm 1 finds an optimal solution for problem P1 in $O(n^{3f+1} f^2)$ time.

Proof. Algorithm 1 proceeds according to the forward state transition mode. The dynamic program exploits properties of an optimal schedule from lemma 1 ,so the algorithm eventually finds an optimal solution. The ranges of the values l_i , x_i , y_i , b and j are $0 \le l_i \le n_i$, $0 \le x_i \le l_i$, $0 \le y_i \le l_i$, $i = 1, 2, \cdots, f$; $1 \le b \le n$, and $1 \le j \le f$, respectively, so the number of possible values for the state variables is $O(n^{3f+1} f)$. In the recursive relation, we need at most $O(f)$ time to complete the state transitions in the second term. Therefore the algorithm requires an overall computational time of $O(n^{3f+1} f^2)$.

Optimal algorithms for P2

We have the following optimal property for problem P2.

Lemma 2. For problem P2, there exists an optimal schedule such that the on-time jobs within each family are processed and delivered in nondecreasing order of their due dates (EDD), and jobs assigned to one delivery batch are processed consecutively on the machine.

Denote the due date time sequence for family i by $d_{i1} \le d_{i2} \le \cdots \le d_{in_i}$, $i = 1, 2, \cdots, f$. Note that late jobs will not be produce or delivery. Based on the optimal properties stated in lemma 2 ,we develop the following dynamic programming algorithm for P2.

Algorithm 2

Define $V(l_1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; u, j, g)$ as the minimum makespan for processing the on-time jobs that satisfy the following conditions:

(1) the total number of scheduled jobs is $l_1 + \cdots + l_f$,of which l_u jobs are from the top of the EDD sequence for customer u , $u = 1, 2, \cdots, f$;

(2) the number of setups of s_i for job family i is $x_i (x_i \le l_i)$, $i = 1, 2, \cdots, f$;

(3) the number of deliveries of family i is $y_i (y_i \le l_i)$, $i = 1, 2, \cdots, f$;

(4) the first job in the last delivery batch is g ,and the jobs belong to family $j (1 \le j \le f)$;

(5) the total number of late jobs is u , $0 \le u \le n$.

Boundary condition:

$$V(0, \cdots, 0; 0, \cdots, 0; 0, \cdots, 0; 0, 0, 0) = 0 \qquad (4)$$

If $V(l_1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; u, j, g) + t_j > d_{jg}$, and $g > 0$,then define

$$V(l_1, \cdots, l_f; x_1, \cdots, x_f; y_1, \cdots, y_f; u, j, g) = +\infty \qquad (5)$$

Recurrence relation:

$$V(l_1,\cdots,l_f;x_1,\cdots,x_f;y_1,\cdots,y_f;u,j,g)=$$

$$\min\begin{cases} V(l_1,\cdots,l_j-1,\cdots,l_f;x_1,\cdots,x_f;y_1,\cdots,y_f;u-1,j,g); \\ p_{jl_j}+V(l_1,\cdots,l_j-1,\cdots,l_f;x_1,\cdots,x_f;y_1,\cdots,y_f;u,j,g), \\ \text{if } 0<g<l_j, V(l_1,\cdots,l_j-1,\cdots,l_f;x_1,\cdots,x_f;y_1,\cdots,y_f;u,j,g) \\ +p_{jl_j}+t_j\le d_{jg}; \\ \min\limits_{(i,h)\in J}\{p_{jl_j}+V(l_1,\cdots,l_j-1,\cdots,l_f;x_1,\cdots,x_j',\cdots,x_f; \\ y_1,\cdots,y_j-1,\cdots,y_f;u,i,h)\} \qquad \text{if} \quad g=l_j \end{cases} \tag{6}$$

$$J=\{(i,h)\,|\,1\le i\le f;1\le h\le l_j-1 \text{ f } i=j;1\le h\le l_i \text{ if } i\ne j;\ x_j'=x_j \text{if } i=j;x_j'=x_j-1$$
$$\text{if } i\ne j;V(l_1,\cdots,l_j-1,\cdots,l_f;x_1,\cdots,x_j',\cdots,x_f;y_1,\cdots,y_j-1,\cdots,y_f;u,i,h)+p_{jl_j}+t_j\le d_{jl_j}\}$$

Optimal solution value:

$$\min\{\omega u+(1-\omega)\sum_{i=1}^{f}y_ic_i\,\Big|\,0\le u\le n,0\le x_j\le n_j,0\le y_j\le n_j,1\le j\le f;$$
$$\{V(n_1,\cdots,n_f;x_1,\cdots,x_f;y_1,\cdots,y_f;u,j,g)\}<\infty$$

Theorem2. Algorithm 2 finds an optimal solution for problem P2 in $O(n^{3f+2}f)$ time.

Proof. Algorithm 2 proceeds according to the forward state transition mode. The dynamic program exploits properties of an optimal schedule from lemma 2 ,so the algorithm finally finds an optimal solution. The ranges of all the values l_i , x_i , y_i , u , g and j are $0\le l_i\le n_i$, $0\le x_i\le l_i$, $0\le y_i\le l_i$, $0\le g\le l_i$, $i=1,2,\cdots,f$; $0\le u\le n$,and $1\le j\le f$, respectively, so the number of possible values for the state variables is $O(n^{3f+2}f)$. In the recursive relation, we need at most $O(nf)$ time to complete the state transitions in the third term, and this state variables is $O(n^{3f+1})$. Therefore the algorithm requires an overall computational time of $O(n^{3f+2}f)$.

Ⅳ. Conclusions

In this paper we consider scheduling with family setups and delivery to multiple customers. We consider two due date related objectives: the first is to minimize the weighted sum of the maximum lateness of jobs to customers and the delivery cost. The second objective is to minimize the weighted sum of the number of late jobs and the delivery cost . We give two optimal algorithm to solve the two problems. Some research topics remain open for future investigation. First, the problems with other customer-related objectives such as total tardiness of the jobs may be studied. The second research topic is to consider the case of processing jobs on parallel machines and delivering them to multiple customers.

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Background of Authors

Lei Wang is a PhD student at the Department of Business Administration, Jinan University, Guangzhou, China. He obtained a Bachelor degree in Mathematics from Shandong University, and a Master degree in Operations Research from The Qufu normal University. His research interests are in supply chain management and machine scheduling.

Guoqing Wang is a Professor at the Department of Business Administration, Jinan University, Guangzhou, China. He obtained a Bachelor degree in Mechanical Engineering and a Master degree in Engineering Management from Harbin Institute of Technology, and a Ph.D. in Operations Research from The Hong Kong Polytechnic University. His research interests are in production and operations management and machine scheduling.